Network Connectivity of One-Dimensional MANETs with Random Waypoint Movement

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***Abstract*— An analysis of network connectivity of one-dimensional Mobile Ad hoc Networks with a particular mobility scheme is presented, focusing on the random waypoint mobility scheme. The numerical results are verified using simulation to show their accuracy under practical network conditions. Observations on RWP properties further lead to approximations and an eventual simple network connectivity formula.**

***Index Terms*— Wireless LAN, land mobile radio data communication, computer network performance.**

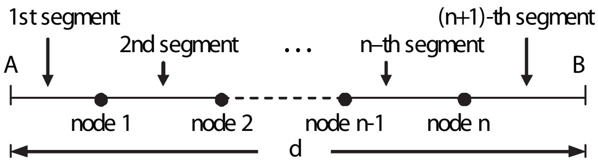

Fig. 1. One dimensional MANET model.

I. Introduction

THE network connectivity of Mobile Ad hoc Networks (MANETs) is a multifaceted problem due to the uncertainty of the network topology. A number of studies concerning network connectivity modeling and analysis of a particular MANET have been reported in the recent technical literature [1]-[6]. All these research works aim at providing a mean to calculate the *Grade of Services* (GoS) measured in network connectivity probability. One of the commonly considered MANETs is the one-dimensional (1-D) MANET which consists of a collection of statistical identical mobile nodes randomly located on a line (not necessarily straight). Some examples of such a 1-D MANET are a vehicular MANET built along a highway in a city environment [7],[8], and a MANET deployed along an attack route in battlefields.

In this letter, we analyze the 1-D MANET connectivity probability, that is the probability that the source/destination pair (shown in Fig. 1 as nodes A and B) of a 1-D MANET can communicate with each other given a certain number of mobile nodes moving according to a particular mobility scheme along the line between the source and the destination. Here, we focus on the random waypoint (RWP) mobility scheme [9]. Such an analysis will be useful for certain applications such as BusNet [7] where buses are mobile and bus stops are stationary.

Works closely related to this research are as follows. Santi *et al.* [1] study the minimum number of nodes required to ensure that a 1-D MANET is "most likely" to be fully connected. Both Bettstetter [3] and Dousse *et al.* [4] study the network connectivity of a 1-D MANET with assumptions that the network size is very large and mobile nodes are placed according to a Poisson point process with certain finite intensity of network density. In [5], Desai and Manjunath derive a network connectivity formula for a 1-D MANET with uniform placement of nodes within a finite range. With a more precise description on the distance between neighboring nodes, Foh and Lee [6] develop a simple closed form network connectivity formula for a 1-D MANET with uniform placement. A MANET connectivity analysis considering RWP is given by Bettstetter in [10]. Here, we take a different approach to derive the 1-D MANET connectivity probability with a general mobility scheme, and we demonstrate with RWP in this letter.

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II. Network Connectivity Analysis

Our considered 1-D MANET is shown in Fig. 1. Three key parameters describing the 1-D MANET are: 1) the network size, d: the distance between the source and the destination, normalized to the radio range of a mobile node; 2) the network density, n: the number of nodes moving between the source and the destination; and 3) the mobility scheme of the n nodes. The source and the destination are stationary and located at the two different edges of the MANET. With this configuration, there are $n+1$ *segments* formed in the MANET.

To begin with, we define a few quantities as follows:

- X = A random variable (r.v.) describing the location (precisely, the distance away from node A) of a particular mobile node at an arbitrary instant of time. The probability density function (pdf) and the cumulative distribution function (cdf) of X are $f(\cdot)$ and $F(\cdot)$ respectively.
- K_x = A r.v. describing the length of a segment with its left edge located at x. The pdf and cdf of K_x are $q_x(\cdot)$ and $Q_x(\cdot)$ respectively.
- L_j = A r.v. describing the location of the left edge of the j-th segment. Let $r_j(\cdot)$ be the pdf of L_j.
- $s_{j,k}$ = The probability that the j-th segment has a length shorter than or equal to k. In this application, k is a constant describing the radio range, i.e. $k = 1$.

$P_n(d)$= The probability that a 1-D MANET of two nodes, with a distance of d and n additional nodes within the two nodes, is connected.

Similar to [4]-[6], our considered 1-D MANET is fully connected at a particular time if all segments have a length that is shorter than or equal to k. We first derive $q_x(k)$ to be

$$q_x\left(k\right)=nf\left(x+k\right)\left[1-\left(F\left(x+k\right)-F\left(x\right)\right)\right]^{n-1} \quad (1)$$

where $0 \le x \le d$ and $0 \le k \le d-x$. The above result is derived based on the concept that a segment of length k with its left edge located at x will be formed if, taking location x as a reference point, one of the n nodes is found at location $x+k$ and all the other $n-1$ nodes are located outside the range between x and $x+k$. The cdf of K_x is thus

$$Q_x\left(k\right)=1-\left[1-\left(F\left(x+k\right)-F\left(x\right)\right)\right]^{n}. \quad (2)$$

The left edge of the j-th segment will be located at l if one node is located at l, and the remaining $j-2$ and $n-j+1$ nodes are found on the left and right sides, respectively, of the node located at l. Hence, we obtain

$$r_j\left(l\right)=nf\left(l\right)\binom{n-1}{j-2}\left[F\left(l\right)\right]^{j-2}\left[1-F\left(l\right)\right]^{n-j+1} \quad (3)$$

with $2 \le j \le n+1$, $0 \le l \le d$, and $\binom{n-1}{j-2}$ the binomial coefficient.

The probability that the j-th segment with its left edge located at x has a length shorter than or equal to k can be expressed as $Q_x(k) \cdot r_j(x)$. This expression is an approximation, however, we will demonstrate with the final numerical results that when n is large, the accuracy is reached. With this approximation, $s_{j,k}$ is derive by unconditioning the expression $Q_x(k) \cdot r_j(x)$ with respect to x. That is

$$s_{j,k}=\int_0^{d-k}Q_x\left(k\right)r_j\left(x\right)dx+\int_{d-k}^{d}r_j\left(x\right)dx. \quad (4)$$

In (4), due to the border effect, when the left edge of a segment falls between $d-k$ and d, the segment is always shorter than k, indicating a connection, i.e. $Q_x(k)=1$.

As indicated in [6], the statistical characteristics of each segment length are approximately independent and identical. Hence knowing that the first segment is always located at node A, and the minimum number of nodes required to achieve a connection between nodes A and B is $d-1$, we get

$$P_n\left(d\right)=\begin{cases}0, & n<d-1\\ Q_0\left(k\right)\left(\prod_{j=2}^{n+1}s_{j,k}\right), & n\ge d-1\end{cases}. \quad (5)$$

With the above result, if we consider uniform placement of nodes, that is $f(x)=1/d$, (5) is reduced to that of [6] with an eventual closed form expression for $P_n(d)$ when ignoring the border effect in (4), which is

$$P_n\left(d\right)=\left(1-\left(1-\frac{1}{d}\right)^{n}\right)^{n+1}, n\ge d-1. \quad (6)$$

III. Network Connectivity Probability with RWP

For the RWP mobility scheme, we note that the analysis of the steady-state node distribution of the RWP mobility scheme is given in [9], with pdf and cdf shown as follows.

$$\begin{cases}f\left(x\right)=-\frac{6}{d^3}x^2+\frac{6}{d^2}x, 0\le x\le d\\ F\left(x\right)=-\frac{2}{d^3}x^3+\frac{3}{d^2}x^2, 0\le x\le d\end{cases}. \quad (7)$$

Using (7) and (2)-(5), with a numerical solver, we compute the probability that the source/destination pair with d distance apart is connected given n nodes moving along the line between the source/destination pair according to RWP.

However, our observations on the RWP properties suggest that further approximations can be considered to simplify (5). We shall only report our observations, investigations on the accuracy of the approximations are not studied in this letter.

We first observe that the variance of L_j is small. Thus, by approximating $r_j(l)$ as a delta function, $\delta(l)$ with $l=E[L_j]$, where $E[L_j]$ is the mean of L_j, unconditioning $Q_x(k)\cdot r_j(x)$ can be expressed by $Q_x(k)$ where $x=E[L_j]$. Moreover, we ignore the border effect at the right edge, then in (4), $s_{j,k}=Q_x(k)$ with $x=E[L_j]$. We will discuss later that the border effect has little influence to the results due to the symmetrical node distribution of RWP about the center.

A closer evaluation on $E[L_j]$, as a function of j, further suggests that $E[L_j]$ is approximately a linear function that can be described by $\frac{j\cdot d}{(n+1)}$. The impact of this approximation is not high as $Q_x(k)$ gives similar values around a particular x.

We also observe that $f(x)$ in (7) is symmetric about $d/2$. Hence the first and the $(n+1)$-th segments share the same statistical property, so are the second and the n-th segments, the third and the $(n-1)$-th segments, and so on. In other words, the connectivity probabilities of the first half segments counting from the left are statistically identical to that of the second half, we use the first half segment connectivity probabilities to compute that of the second half, and hence the border effect at the right edge of the 1-D MANET can be ignored. With these approximations, we simplify (5) to be

$$P_n\left(d\right)=\begin{cases}0, & n<d-1\\ \left(Q_0\left(k\right)\prod_{j=2}^{m}Q_c\left(k\right)\right)^2, & n\ge d-1\end{cases} \quad (8)$$

where $k=1$, $m=\lfloor(n+1)/2\rfloor$ and $c=\frac{j\cdot d}{(n+1)}$. Strictly speaking, the above result should be expressed differently for even and odd values of n. However, aiming to develop a simple formula, noticing that the center of the network is with high density of nodes due to the RWP properties, it is not critical to neglect the connectivity probability of a segment around the center, which usually gives a connectivity probability closed to one when n is large.

IV. Numerical Results

In Fig. 2, the numerical results of network size versus network connectivity probability with different densities given by (5) (shown with thick solid lines) are plotted and compared with the simulation results (shown with symbols). Our simulation is an event-driven program written in C++. The simulated

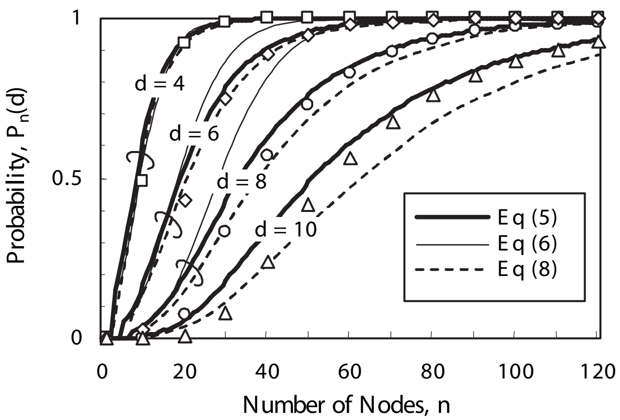

Fig. 2. MANET connectivity probability vs. number of nodes.

RWP mobility scheme follows that of [9] where all n nodes are moving with no pause time along a straight line linking the source and the destination. The simulation program measures the proportion of time that the network is fully connected. Simulation results are obtained with a 95% confidence interval with radius less than 0.002.

The comparison presented in Fig. 2 indicates that our numerical results given by (5) produce a slightly higher value of $P_n(d)$ than the actual one obtained from the simulation given a particular set of n and d values. However, the difference between the numerical and simulation results becomes small when the desired network connectivity probability is high (e.g. over 0.9). Since a practical network usually assumes a high value of $P_n(d)$, this allows our formula to be practically used in the network design.

To verify the accuracy of the approximated results given by (8), we compare the approximated results (shown with dotted lines in Fig. 2) with the simulation results. It is found that approximated numerical results give good estimation as well when the desired $P_n(d)$ is high (over 0.95 in this case).

To illustrate the impact of mobility schemes, we plot the network connectivity probabilities of uniform placement (shown with thin lines) for $d = 4, 6$ and 8 in Fig. 2. It can be seen from the results that, generally, the need for the number of nodes can be of great difference for different mobility schemes given a certain $P_n(d)$ and a large d value. However, interestingly, for $d = 4$, the differences between the network connectivity probabilities of RWP and uniform placement of nodes are hardly noticeable. This suggests that the actual mobility scheme in a 1-D MANET of small coverage (i.e. a small d value) may not have a significant impact on the physical network connectivity and that (6) may be used as a good estimator in the network design.

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