A Closed Form Network Connectivity Formula for One-Dimensional MANETs

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***Abstract*— In this paper, a closed form network connectivity formula for a one-dimensional Mobile Ad hoc Network (MANET) is developed. Precisely, we derive the probability that a MANET is fully connected given a certain number of nodes randomly and uniformly placed along a path between a source and destination pair. This formula is particularly useful in the process of design and deployment of a MANET. The formula also provides a critical constraint function to the problem of network optimization. It formulates the relationship between the number of mobile nodes required for a particular MANET given a desired network connectivity probability. An approximation is employed to achieve the final closed form expression. The approximation is then tested by simulation to show the accuracy of our formula under practical network conditions.**

I. INTRODUCTION

A wireless mobile ad hoc network (MANET) is one of a practical solution to rapidly setup a network in an area where infrastructure networks do not already exist. A MANET is a collection of mobile nodes that communicate with each other without a centralized control. Each node is responsible for forwarding a packet it has received from one to another if required, until the packet reaches the destination. The network topology may change from time to time due to the mobility of the nodes. Because of the uncertainty of the network topology, many performance results developed from the analytical works focusing on fixed or infrastructure networks are not applicable directly to MANETs.

There are several studies in the literature focusing on the network connectivity problem in MANETs. Some important studies are the study of optimum transmission radii by Kleinrock et al. [1], the study of channel capacity of MANETs by Gupta et al. [2] and other works extending [2], as well as the study of network connectivity for a particular MANET [3]-[5].

In this paper, we focus on the study of a one-dimensional (1-D) MANET. Specifically, a MANET that consists of a collection of statistical identical nodes randomly placed on a line (not necessarily straight). This type of MANETs does appear in many real world network applications, for example, a vehicular MANET built along a highway in a city environment [7]-[8], a MANET deployed along an attack route in battlefields [9], and others that align nodes in a line.

We derive the probability that a source and destination pair is connected given that they are at a distance apart from each other with some number of nodes placed uniformly and randomly in between. Some of the most related work to ours presented in the literature were given in [3]-[5]. Santi et al. [3] study the minimum number of nodes required to ensure that a 1-D MANET is "most likely" to be fully connected, however, the actual probability of network connectivity is not indicated. Both Bettstetter [5] and Dousse et al. [4] also study the network connectivity of a 1-D MANET. The authors consider that the network size is very large and mobile nodes are placed according to a Poisson point process with certain finite intensity of network density. However, the assumption network configuration is hard to be translated to a practical network environment, and the application of the developed results is not straightforward.

In this paper, we study the network connectivity of a 1-D MANET given a certain distance between the source and destination pair with a certain number of mobile nodes placing in between. Our result improves [3] by providing the "exact" probability of network connectivity (note that both the considered 1-D MANETs are slightly different), and our work also generalizes the work given in [4]-[5] where their work is a special case of ours when the distance between the source and destination pair is set to a very large value. Most importantly, based on an observation from our model, we introduce an approximation that significantly simplifies the network connectivity formula leading to a convenient closed form expression, which is later shown to be very accurate. This closed form result is also an important constraint function for the network optimization. The result enables a network designer to compute precisely the minimum number of mobile nodes required to meet a certain *Grade of Service* (GoS) measured by the probability of network connectivity.

In the next section, we present the analysis of the network connectivity of a 1-D MANET. Analytical results are studied in Section III, and some important conclusions are drawn in Section IV.

II. MANET CONNECTIVITY ANALYSIS

In our considered MANET, a node, say, node i, can communicate directly with another node, say, node j, only if the distance along the line between nodes i and j is less than or

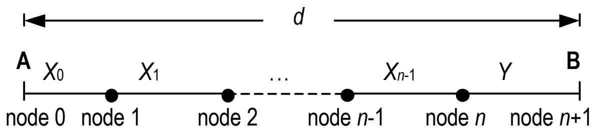

Fig. 1. One-dimensional MANET model.

equal to the radius of the radio range, r. However, if node j is located outside the radio range of node i, they can communicate indirectly if there exist some intermediate nodes between them providing a series of direct communications through the intermediate nodes. A MANET is said to be connected if all nodes communicate with each other directly or indirectly.

Our analysis focuses on the network connectivity of the considered MANET. We develop a closed form result of *1-D MANET connectivity formula* which describes the probability that two nodes A and B, with a distance of d apart from each other along a line, is connected given an additional of n nodes uniformly and randomly placed between the nodes A and B as shown in Fig. 1. In the following analysis, we assume that the radius of the radio range, r, is of one unit distance long, and the distance variable d is normalized to r. The variables d and n can also be interpreted as the *network size* and the *network density* respectively.

We begin by defining node 0 to be node A, node 1 to be the node closest to the right side of node 0, node 2 to be the node closest to the right side of node 1, and so on (see Fig. 1). Let X_i be the random variable of the distance between node i and node i+1, i=0,1,2,…,n–1, and Y be the random variable of the distance between node n and node B. Similar to the technique used in [4], nodes A and B are connected only if $X_i \le 1$, i=0,1,2,…,n–1 and $Y \le 1$.

Define $q_n(k)$ to be the probability density function of X_i, i=0,1,2,…,n–1, that is, the distance between nodes i and i+1 is k. A distance k is formed between neighboring nodes i and i+1 if node i is placed at a particular location, say, z away from node A, and all other nodes are placed outside the segment between z and $z+k$. Due to the uniform placement of all intermediate nodes, ignoring the border effect, we get

$$q_n(k) = n\left(\frac{1}{d}\right)\left(1-\frac{k}{d}\right)^{n-1}, \quad 0 \le k \le d. \tag{1}$$

Let $Q_n(k)$ be the cumulative distribution function of X_i. Using the previous result, we obtain

$$Q_n(k) = 1-\left(1-\frac{k}{d}\right)^{n}, \quad 0 \le k \le d. \tag{2}$$

Note that if we take $n,d \to \infty$, then $Q_n(k)$ becomes an exponential random distribution function which is the case of [4],[5].

Let $p(i,j)$ denote the probability that nodes i and j are connected, to obtain the probability that nodes A and B are connected, i.e. $p(0,n+1)$, we first obtain the following recursive formula [4]

$$p(0,n+1) = \int_0^1 q_n(x)\, p(1,n+1)\, dx. \tag{3}$$

The above formula states that nodes 0 and n+1 are connected if the distance between nodes 0 and 1 is less than or equal to the radio range, r (which is assumed to be one unit distance), implying a connection between nodes 0 and 1, and at the same time, node 1 is somehow connected to node n+1.

By expanding the above result recursively, we get

$$\begin{aligned} p(0,n+1) = {} & \int_0^1 q_n(x_0) \int_0^1 q_n(x_1) \cdots \\ & \int_0^1 q_n(x_{n-1}) f(y)\, dx_{n-1} \cdots dx_1 dx_0 \end{aligned} \tag{4}$$

where

$$f(y) = \begin{cases} 1, & \text{if } y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

and

$$y = d - \sum_{i=0}^{n-1} x_i$$

The function $f(y)$ is the probability that the rightmost connection of the 1-D MANET shown in Fig. 1, described by Y, is connected. In other words, it is the probability that nodes n and n+1 are connected, that is $f(y) = \Pr\{Y \le 1\}$, where the random variable Y is given by

$$Y = d - \sum_{i=0}^{n-1} X_i \,. \tag{5}$$

To obtain a fully connected 1-D MANET, we must have $X_i \le 1$, i=0,1,2,…,n–1, and $f(y) = 1$. If $n < d-1$, then $f(y)=0$, as the number of nodes is insufficient to form a connection between nodes A and B, hence a fully connected network cannot be resulted. When $n \ge d-1$, we observe from Fig. 1 that the random variable Y may be seen as an independent random variable behaving like X_0, especially when $n >> d$. Hence when $n >> d$, we can expect $\Pr\{Y \le 1\}$ to be similar to $\Pr\{X_0 \le 1\}$, and Y to be less dependent on X_i random variables, i=0,1,2,…,n–1. If we approximate Y to be an independent random variable identical to X_0, that is

$$\Pr\{Y \le 1\} = \int_0^1 q_n(x)\, dx\,, \quad n \ge d-1\,, \tag{6}$$

equation (4) can be simplified into

$$p(0,n+1)=\begin{cases}0, & n<d-1\\ \left(\int_0^1 q_n(x)dx\right)^{n+1}, & n\geq d-1\,.\end{cases} \tag{7}$$

Since the distance between node 0 and n+1 is d, we rewrite the previous result as

$$P_n(d)=\begin{cases}0, & n<d-1\\ \left(1-\left(1-\frac{1}{d}\right)^n\right)^{n+1}, & n\geq d-1\end{cases} \tag{8}$$

where the above result is the *1-D MANET connectivity formula* with parameters d and n. Precisely, $P_n(d)$ is the probability that a one-dimensional MANET of two nodes with a distance of d and n additional nodes randomly and uniformly placed within the MANET is connected.

Under the condition that $n \geq d-1$, based on (8), we can easily obtain the relationship between the network size, d, and the network density, n, given a certain network connectivity probability. Let $s = P_n(d)$, after some algebraic manipulations, we get the following relationship

$$d=\left[1-\left(1-s^{\frac{1}{n+1}}\right)^{\frac{1}{n}}\right]^{-1}. \tag{9}$$

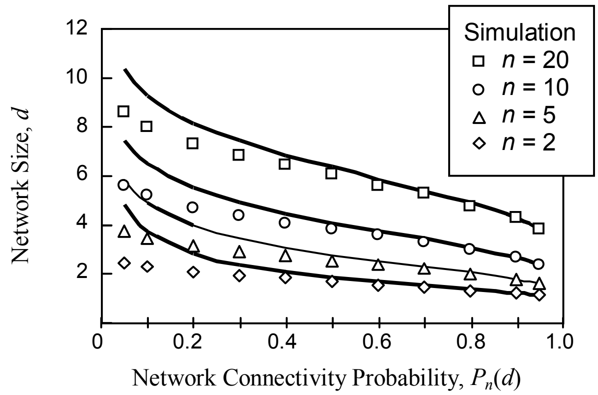

Fig. 2. Network size versus network connectivity probability.

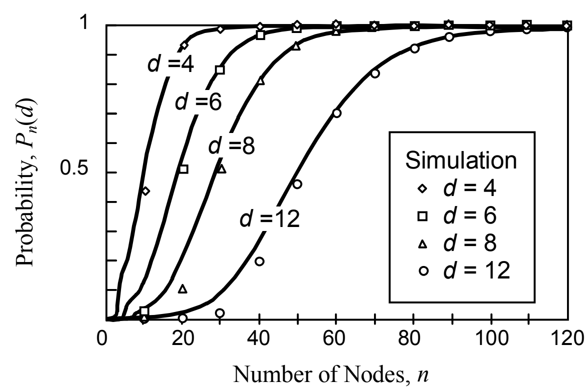

Fig. 3. Network connectivity probability versus number of nodes.

III. Analytical Results

Our 1-D MANET connectivity formula given in (8) is derived based on the assumption that Y is identical to X_0 shown in Fig. 1. Testing of the analytical results is necessary to justify the employed approximation. We compare the analytical results with the simulation results to demonstrate the accuracy of our analysis. The following subsections present studies of the MANET connectivity from different aspects.

A. Test of the Approximation

To show the impact of the employed approximation, in Fig. 2, the results of network size, d, versus network connectivity probability, $P_n(d)$, with different network density, n, are plotted. The solid lines in the figure represent analytical results and symbols represent simulation results. The analytical results are given by (8). By comparing the analytical and simulation results, we notice that when the network connectivity probability is low, our formula overestimates the network size given a certain network connectivity probability and network density, that is, to maintain a certain network connectivity probability with a given network density, the actual network size should be smaller than that suggested by our formula. However, as the network connection probability increases, the gap between the analytical and simulation results reduces, particularly when $P_n(d)$ is larger than 60%. The analytical results become very accurate when $P_n(d)$ reaches over 80%.

The high accuracy of our formula at a high region of network connectivity probability is important because the high network connectivity probability is usually a requirement for a practical network design. In addition, a snapshot of the node distribution of a network providing full connectivity does not always lead to a successful packet delivery. This connectivity is the minimum requirement for a packet delivery, whereas the success of a packet delivery will also depend on whether the network remains connected during the packet forwarding process. Hence a high network connectivity probability is often needed in the network design to ensure the successful packet forwarding, not just the network connectivity in a snapshot. This allows our formula, which gives accurate results when the probability of network connectivity is high, to be practically used in the design of a 1-D MANET.

B. Impact of Network Density on MANET Connectivity

Fig. 3 plots the analytical (shown with solid lines) and simulation (shown with symbols) results of network connectivity probability versus the number of nodes with different network sizes. We again notice a good match between

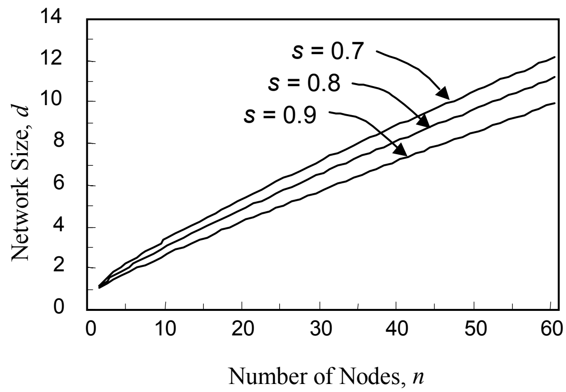

Fig. 4. Network size versus network density.

the numerical and simulation results, especially when the probability of network connectivity is high.

We observe that, given a high network connectivity probability, $P_n(d)$, a small change in the probability may result in a large change in the requirement of the number of nodes. The number of nodes plays an important role in the network design as it decides the cost of a network, our formula provides an important constraint function in the network design optimization process. Precisely, it provides the relationship between the cost of a MANET and the GoS measured by the network connectivity probability.

C. Network Size versus Network Density

In this subsection, we study the relationship between the network size and the network density based on the developed result given in (9). Fig. 4 presents the network size versus network density with different network connectivity probabilities. Notice the differences between the presented curves, these differences indicate that a different required GoS will need a different network density given a certain network size. Hence knowing the exact probability of network connectivity is crucial to achieve a more precise network design. With this result, we overcome the drawback of the work presented in [3].

The results also show that the network size appears to grow linearly as the number of nodes increases. In fact, the network density grows slightly faster than the network size, this again confirms the authors' claim that a large-scale 1-D MANET (i.e. $d \to \infty$) will surely be unconnected [4]. In addition, if we know the network size, our result will provide the exact probability that the network is fully connected.

IV. Conclusions

In this paper, we have developed a convenient yet accurate 1-D MANET connectivity formula. Precisely, we derive the probability that a 1-D MANET is connected given a certain distance between a source and a destination with some number of nodes randomly placed in between. An approximation was employed and was later shown to be accurate under the condition where our formula is likely to be used. The relationship between the network size and the network density given a desired GoS measured by the network connectivity probability was also formulated. Our closed form expression has made the optimization of a MANET design possible. An immediate application of our formula is the design of a BUSNET [7] where buses (as mobile nodes) and bus stops (as stationary nodes) are used to build a metropolitan area MANET. The network connectivity for such a MANET can be analyzed using our developed formula to achieve a better network planning and deployment.

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