A New Technique for Performance Evaluation of Random Access Protocols

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***Abstract* – A new technique is proposed for performance evaluation of random access protocols. This technique is based on the idea that the service probability distribution of a particular random access protocol can be described by a phase-type distribution. By modeling the service probability distribution of a protocol into a phase-type distribution, the performance of the protocol can be evaluated using an equivalent Continuous Time Markov Chain. In this paper we demonstrate the applicability of the technique to performance evaluation of the IEEE 802.3 and the IEEE 802.11 MAC protocols. Furthermore, we provide an example of performance evaluation of a MAC protocol subject to bursty traffic models such as the Markov Modulated Poisson Process. The accuracy the results are verified by simulation.**

I. INTRODUCTION

Performance evaluation of random access protocols such as Aloha, Carrier Sense Multiple Access (CSMA), CSMA with Collision Detection (CSMA/CD), CSMA with Collision Avoidance (CSMA/CA) has been for many years a popular research topic (see [1,2] and references therein). Typically, because the different protocols differ in their operations, these protocols were analyzed separately. By comparison, this paper provides a unified approach that can be applicable for performance evaluation of various different protocols.

Because of the complexity of these protocols, only simple traffic conditions such as Poisson have been assumed [3-5]. In many cases, further simplifying assumptions on the protocol operation and/or the retransmission algorithm must been made in the analyses. Hence, the scope of the results of such performance analyses may be limited.

A saturation traffic condition is considered in some random access protocol analyses [2,6,7]. Under this condition, all stations are assumed to be *saturated* such that whenever they successfully complete a data frame transmission, they are immediately ready for another data frame transmission. During the period when a data frame is being served, no additional arrival appears. Due to its deterministic nature, the saturation throughput analysis is often achievable with an accurate model describing the protocol operations.

However, the value of the results obtained under the saturation traffic condition is again limited because it does not provide insight into the performance of a protocol under statistical traffic conditions. In this paper, we propose a new technique for performance evaluation of such protocols. The technique offers accurate performance evaluation and under the assumption of more general traffic models such as the Markov Modulated Poisson Process (MMPP) without the need for simplifying assumptions on the protocol operations.

Our technique can be briefly described as follows. Consider k stations accessing the medium according to a certain random access protocol. The k stations are subject to bursty traffic demand so that at each point in time, only i stations are active. Further assume that we also know the probability distribution of the time between two successive successful data frame transmissions for a given number of active stations and no additional station becomes active during that period of time. In this case, we can model the system by a Continuous Time Markov Chain Single Server Queue (CTMC-SSQ). The CTMC-SSQ can be a state-dependent, multi-dimensional Continuous Time Markov Chain.

The "arrival" process of this CTMC-SSQ represents the process of idle stations that become active. The "service" time is the time it takes to transmit a data frame successfully. The statistical characteristics of the service time for each i are given by the saturation throughput analysis and can be fitted with an equivalent phase type (PH) distribution [8], so that a CTMC-SSQ structure can be used.

The main advantages of our proposed technique are: (i) it takes advantage of saturation throughput analyses which are often achievable and accurate, and reuses those accurate results for performance evaluation under statistical traffic; (ii) having obtained the PH statistics for each i, we no longer need to consider the details of the actual protocol which significantly simplifies the analysis; (iii) we can take advantage of the vast knowledge available on analysis of Continuous Time Markov Chains including Neuts' matrix geometric solutions [8]. The disadvantage of the technique is that it relies on numerical solutions, and does not lead to closed form results.

The paper is structured as follows. In the next section, our proposed technique is described in detail. In Section III, we apply the technique to two MAC protocols. Simulation results are presented to demonstrate the accuracy of the technique. Section IV demonstrates the power of the technique by analyzing the IEEE 802.11 MAC protocol under a more bursty traffic model – the MMPP.

II. THE TECHNIQUE

We consider k stations sharing a transmission medium. They are fed by a certain arrival process and try to access the medium according to a given random access protocol. The aim is to obtain numerical values for certain performance measures such as the mean queueing delay.

We begin by describing a saturation throughput analysis of a protocol, whereby we assume that there are only i stations sharing a transmission medium, $1 \le i \le k$, and all i stations are saturated so that whenever a station successfully transmits its data frame, the station is ready for another new data frame transmission immediately after the previous data frame is successfully transmitted. In other words, all i stations are always active. The main output from the analysis is the saturation throughput of the protocol given i saturated stations, $S(i)$.

From the saturation throughput, the mean service rate of a protocol given i active stations, $\mu(i)$, can be obtained by:

$$\mu(i) = \frac{S(i)}{t_d} \qquad (1)$$

where t_d is the mean transmission time of the payload in a data frame. In another words, $1/\mu(i)$ is the mean time between two successive successful data frame transmissions given i saturated stations. Notice that it is the sum of (i) the channel assignment time [3], (ii) the mean header transmission time in a data frame, and (iii) the mean transmission time of the payload in a data frame, t_d.

To analyze the protocol under a statistical traffic, we construct a CTMC-SSQ with a certain arrival and service processes. For the purpose of illustrating the technique, we consider a simple example of a CTMC-SSQ, the *state dependent* M/M/1/k (SD-M/M/1/k) queue. Under this model, there are k stations in a network. The *state* of the queue represents the number of active stations. The number of active stations increases according to a Poisson process with parameter λ, and it decreases based on the state dependent service process.

From the saturation throughput analysis, the mean service rate given i active stations is obtained by (1). Since the state in the SD-M/M/1/k queue represents the number of active stations, together with the service rates obtained from the saturation throughput analysis, then the SD-M/M/1/k system can be used to analyze the protocol. Its transition state diagram is shown in Fig. 1. The use of the SD-M/M/1/k system implies the assumption that the service time under saturation for any number of saturated stations is exponentially distributed. In many cases, this assumption may not be accurate. In such a case, we propose to perform a more accurate fit of the statistical characteristics of the service time under saturation and service time of our CTMC-SSQ model, for each i where $1 \le i \le k$. To maintain the Markov chain structure, a PH distribution [8] (e.g. the Erlang distribution) could be used.

For more realistic performance evaluation, it may be required to analyze a protocol with an arrival process that is more bursty than a Poisson process. Any arrival process falls under the versatile *Markovian Arrival Process* (MAP) can be used. One such example is the MMPP. In Section IV, we will demonstrate the performance evaluation of a MAC protocol where the process of idle stations that become active follows the MMPP.

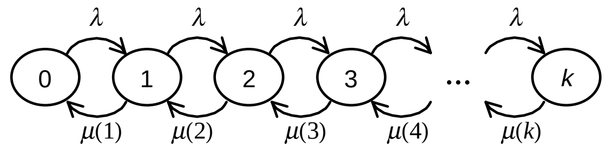

Fig. 1. The state dependent service rates M/M/1/k queue for the analysis of a random access protocol.

III. NUMERICAL EXAMPLES

In this section, we provide several examples of performance evaluation using our proposed technique. We consider two widely known MAC protocols – the IEEE 802.3 and the IEEE 802.11 MAC. Here we assume that the process of the increase in the number of active stations is a Poisson process, and the mean data frame delay is the performance measure.

To analyze the protocols using our proposed technique, the saturation throughput analysis must be obtained first. The saturation throughput of the IEEE 802.11 and the IEEE 802.3 MAC protocols are given in [2] and [6] respectively. In particular, the $S(i)$ values for the IEEE 802.11 MAC protocol are given in [2], equation (7). The $S(i)$ values for the IEEE 802.3 MAC protocol are given in [6], equations (1) and (9). Having these $S(i)$ values, the $\mu(i)$ values can be calculated by (1). We will now follow the assumptions in [2,6] except that the input traffic is now statistical (based on a Poisson process) instead of saturated.

A. *Performance Evaluation of IEEE 802.11*

We first demonstrate the technique for the IEEE 802.11 MAC protocol with k=50, that is the maximum number of stations in the network is 50. The Cumulative Distribution Function (CDF) of a typical service time distribution of the IEEE 802.11 MAC protocol is shown in Fig. 2, with the protocol parameters shown in Table 1. Fig. 2 also compares that CDF with the CDFs of exponential, Erlang with parameters 8 (E_8) and Erlang with parameter 32 (E_{32}) random variables, all with the same mean value. Notice that in Fig. 2, the service time distribution is closed to deterministic. This is because, as per Table 1 and [2], the data frame size is assumed to be fixed and that the payload transmission time, which is also fixed, accounts for just over 80% of the total service time in average. This of course indicates that the IEEE 802.11 MAC protocol is efficient, which is also evident in [2].

As can be seen in Fig. 2, clearly, the service time distribution, under saturated traffic condition, namely, the distribution of the time between two successive successful data frame transmissions, of the IEEE 802.11 MAC protocol, is not exponentially distributed. Whereas, E_8 and E_{32} appear

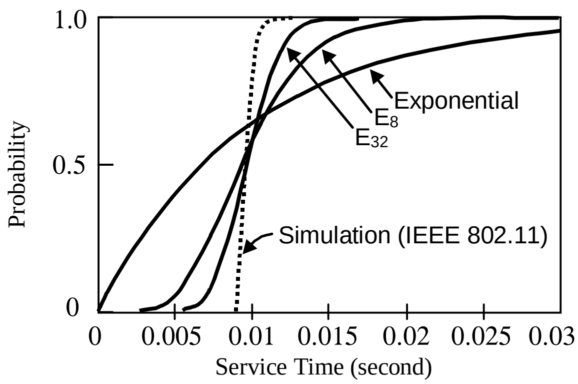

Fig. 2. The CDFs of an exponential, Erlang random variables and the service time of IEEE 802.11 for k=20.

to be better to describe the service time distribution of the protocol.

To analyze IEEE 802.11 with k=50 under Poisson arrivals, we construct three CTMC-SSQs – the SD-M/M/1/50, SD-M/E_8/1/50 and SD-M/E_{32}/1/50 queues, where the prefix SD-represents the state dependent service rates property. The balance equations for the M/M/1/k and M/Ej/1/k queues, and all values to generate results shown in Fig. 3 are given in the Appendix. Parameters used for the protocol are given in Table 1. The numerical solution solved by Successive Over-Relaxation (shown in solid lines) as well as the simulation results (shown in symbols) are compared in Fig. 3.

The setting of k=50 in the M/Ej/1/k queue means that when there are 50 active stations in the network, additional arrivals will be blocked. To have a fair comparison between the numerical and simulation results, the simulation program assumes that no additional arrivals occurs when there is 50 active stations in the network.

The results presented in Fig. 3 indicate that the M/M/1/50 queue underestimates the performance of the protocol. This is mainly because the variance of an exponential service time distribution is higher than the actual one (see Fig. 2), hence the delay obtained from M/M/1/50 is also higher. When Erlang service time distribution is considered, the numerical and simulation results become indistinguishable. It confirms

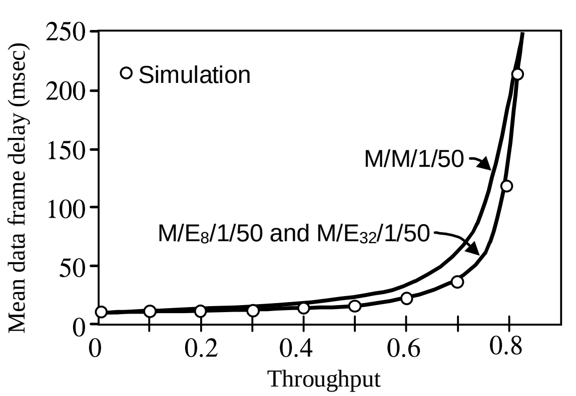

Fig. 3. The delay performance of IEEE 802.11 with different service time distributions.

Table 1. Parameters for the analysis and simulation of the IEEE 802.11 MAC protocol.

Parameter Descriptions	Values
Physical layer parameters	FHSS standard (see [2])
Channel bit rate	1Mb/s
DCF access method	Four-way handshaking
Minimum backoff window	8
Maximum backoff window	256
Payload	8184 bits (constant size)
Maximum station number, k	50
Other protocol parameters	(see [2])

the accuracy of using Erlang random variables for the service time distribution of the IEEE 802.11 MAC protocol, and the reliability of the results obtained by the technique.

Fig. 3 also compares the delay performance of M/E_8/1/50 and M/E_{32}/1/50 queues. In Fig. 2, the CDFs of E_8 and E_{32} are slightly different. However, in the delay performance curves, these two service processes produce almost identical results in Fig. 3. This suggests that E_8 can be used for describing the service process as good as E_{32} for this protocol. Clearly, the use of E_8 reduces the computation time.

B. Performance Evaluation of IEEE 802.3

The saturation throughput of the IEEE 802.3 MAC protocol, used in Ethernet, has been studied in [6]. It follows the protocol model and assumptions developed in [3,9]. Having obtained the service rate, $\mu(i)$, from (1), we construct two CTMC-SSQs – the SD-M/M/1/50 and SD-M/E_8/1/50 queues.

Fig. 4. shows the delay performance of the M/M/1/50 and M/E_8/1/50 queues, as well as the simulation results. The protocol parameters are given in Table 2. As shown in Fig. 4, while the delay curve obtained from the M/M/1/50 underestimates the actual delay performance, the results from M/E_8/1/50 and simulation are closed, which again validates the use of the technique for the analysis of the IEEE 802.3 MAC protocol.

IV. Performance Evaluation of IEEE 802.11 with MMPP Arrival

In this section, we demonstrate the power of the technique by evaluating the performance of the IEEE 802.11 MAC protocol where the process of idle stations that become active follows the MMPP.

A study in [10] shows that LAN traffic, particularly Ethernet, exhibits long range dependent (LRD) property. It is indicated in [11] that although MMPP is a short range dependent (SRD) process, for a limited buffer size queueing system such as LANs, SRD process can be used to model LRD traffic for the purpose of queueing performance evaluation. Hence the bursty MMPP arrival process may be used to model LAN traffic with appropriate parameters.

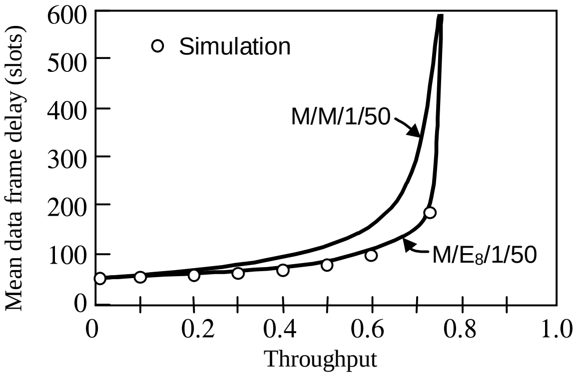

Fig. 4. The delay performance of IEEE 802.3.

However, justification and fitting of MMPP parameters to LAN traffic is out of the scope of this paper.

The IEEE 802.11 MAC protocol with the parameters listed in Table 1 is considered. The SD-MMPP/E_8/1/50 queue is constructed, with balance equations given in the Appendix.

Fig. 5 provides the delay performance obtained by the numerical solutions for the queue and by simulation. As can be seen, excellent agreement between the numerical and simulation results has been reached. It confirms the robustness of our proposed technique.

Moreover, Fig. 5 illustrates the effect of traffic burstiness on the delay performance of the IEEE 802.11 MAC protocol. In Fig. 3, the arrival process is assumed to be a Poisson process. Under such an arrival process, the IEEE 802.11 MAC protocol achieves as high as 80% throughput level. However, when MMPP is considered, as in Fig. 5, only just below 70% can be achieved, and the mean delay started to increase exponentially at a throughput level of 60%. This suggests that the IEEE 802.11 MAC protocol performs quite differently under Poisson and bursty traffic. Therefore the assumption of Poisson arrival traffic may overestimate the performance of the IEEE 802.11 MAC protocol.

V. CONCLUSIONS

In this paper, we have proposed a new technique to analyze

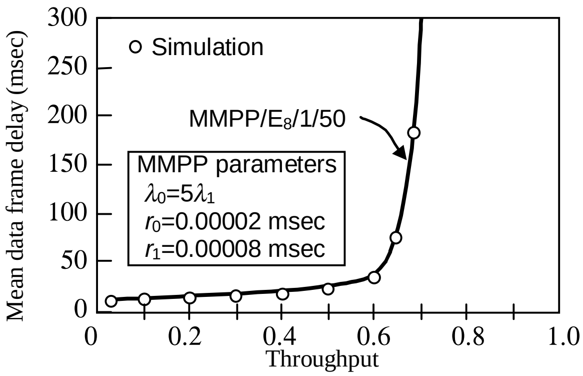

Fig. 5. The delay performance of the IEEE 802.11 MAC protocol under MMPP arrival process.

Table 2. Parameters for the analysis and simulation of IEEE 802.3.

Parameter Descriptions	Values
Channel bit rate	10 Mb/s
Propagation Delay, τ	25.6 µsec
Slot time	2τ
Cost of a collision	one slot
The time required for detecting a transmission end	τ
Payload (headers are ignored)	50slots (constant size)
Normalized propagation delay, α	0.01
Other protocol parameters	(see [10])

a random access protocols. Two widely known MAC protocols were used as examples. Excellent agreement between numerical and simulation results throughout the paper confirms the robustness and accuracy of our proposed technique for random access protocol evaluation.

The main advantage of the technique is that it significantly reduces the analytical overhead of the analysis of a protocol compared to the traditional approach. As a result, the technique allows for a more complex input traffic model and further details of protocol operations to be included in the analysis. Furthermore, much knowledge is available to improve the technique so that more accurate traffic models and useful performance measures can be obtained.

The benefit of the technique was also demonstrated by obtaining performance results for the IEEE 802.11 MAC protocol under bursty (MMPP) traffic conditions without simplifying protocol operation details.

APPENDIX

In this appendix, we provide the steady state balance equations of the M/Ej/1/k and the MMPP/Ej/1/k queues. Note that the M/M/1/k and the MMPP/M/1/k queues are special cases of the M/Ej/1/k and the MMPP/Ej/1/k queues respectively when j=1.

Let p_n be the probability that a system will stay in state n, with $0 \le n \le k$. The mean arrival rate, $\overline{\lambda}$, of a system is:

$$\overline{\lambda} = \sum_{i=0}^{k-1} (\lambda \cdot p_i) \qquad (2)$$

and the throughput, ρ, is given by:

$$\rho = \overline{\lambda} \cdot d_t \qquad (3)$$

where λ is the mean arrival rate of each state, and d_t is the mean transmission time of the payload in a data frame. The steady state balance equations for the M/Ej/1/k queue are:

$$0 = -\lambda p_0 + j\mu(1)\, p_{1,1},$$
$$0 = -(\lambda + j\mu(1))\, p_{1,j} + j\mu(2)\, p_{2,1} + \lambda p_0,$$
$$0 = -(\lambda + j\mu(1))\, p_{1,i} + j\mu(1)\, p_{1,i+1}, \qquad (i = 1,2,\ldots, j-1)$$

$$0=-(\lambda+j\mu(n))p_{n,j}+j\mu(n+1)p_{n+1,1}+\lambda p_{n-1,j}, \quad (n=2,3,\ldots,k-1)$$
$$0=-(\lambda+j\mu(n))p_{n,i}+j\mu(n)p_{n,i+1}+\lambda p_{n-1,i}, \quad (n=2,3,\ldots,k-1;\ i=1,2,\ldots,j-1)$$
$$0=-j\mu(k)p_{k,j}+\lambda p_{k-1,j}, \quad (n=2,3,\ldots,k-1)$$
$$0=-j\mu(k)p_{k,i}+j\mu(k)p_{k,i+1}+\lambda p_{k-1,i}, \quad (i=1,2,\ldots,j-1)$$

with

$$p_n=\sum_{i=1}^{j}p_{n,i}, \qquad (n=1,2,\ldots,k)$$
$$\sum_{n=0}^{k}p_n=1.$$

where $\mu(n)$ is the service rate of a protocol for n saturated stations; p_0 is the probability that the system is in state 0, and $p_{n,i}$ is the probability that the system is in state (n,i).

The parameters used to generate Fig. 3 are: j=1 or 8, k=50, d_t=8.184msec, $\mu(n)$ is obtained by equation (7) in [2] and (1), λ describes the traffic intensity. The mean arrival rate and the throughput of the system can be computed by (2) and (3). The balance equations are solved by Successive Over-Relaxation, and the mean delay is obtained by applying Little's formula.

The parameters for the numerical results presented in Fig. 4 are: j=1 or 8, k=50, d_t=50 slot time, $\mu(n)$ is obtained by, equations (1) and (9) in [6] and (1).

The steady state balance equations for the MMPP/Ej/1/k queue are:

$$0=-(\lambda_m+r_m)p_{0,m}+j\mu(1)p_{1,m,1}+r_{m+1}p_{0,m+1},$$
$$0=-(\lambda_m+j\mu(1)+r_m)p_{1,m,j}+\lambda_m p_{0,m}+j\mu(2)p_{2,m,1}+r_{m+1}p_{1,m+1,j},$$
$$0=-(\lambda_m+j\mu(1)+r_m)p_{1,m,i}+j\mu(1)p_{1,m,i+1}+r_{m+1}p_{1,m+1,i}, \quad (i=1,2,\ldots,j-1)$$
$$0=-(\lambda_m+j\mu(n)+r_m)p_{n,m,j}+j\mu(n+1)p_{n+1,m,1}+\lambda_m p_{n-1,m,j}+r_{m+1}p_{n,m+1,j}, \quad (n=2,3,\ldots,k-1)$$
$$0=-(\lambda_m+j\mu(n)+r_m)p_{n,m,i}+j\mu(n)p_{n,m,i+1}+\lambda_m p_{n-1,m,i}+r_{m+1}p_{n,m+1,i}, \quad (n=2,3,\ldots,k-1;\ i=1,2,\ldots,j-1)$$
$$0=-(j\mu(k)+r_m)p_{k,m,j}+\lambda_m p_{k-1,m,j}+r_m p_{k,m+1,j}, \quad (n=2,3,\ldots,k-1)$$
$$0=-(j\mu(k)+r_m)p_{k,m,i}+j\mu(k)p_{k,m,i+1}+\lambda_m p_{k-1,m,i}+r_{m+1}p_{k,m+1,i}, \quad (i=1,2,\ldots,j-1)$$

with

$$m+1=\begin{cases}1, & if\ m=0\\ 0, & if\ m=1\end{cases}$$
$$p_0=p_{0,0}+p_{0,1}$$
$$p_n=\sum_{i=1}^{j}\left(p_{n,0,i}+p_{n,1,i}\right), \qquad (n=1,2,\ldots,k)$$
$$\sum_{n=0}^{k}p_n=1.$$

where m=0 or 1 is the mode of MMPP, λ_m is the mean arrival rate in mode m of MMPP, $\mu(n)$ is the service rate of a protocol for n saturated stations.

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